How is China's coke price related with the world oil price? The role of extreme movements

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1. Introduction

As a crucial raw material in the iron and steel industries, coke has been fueling the engine of China's economic boom economy for the past decade. Domestic coke consumption in China surged from around 108 million metric tons (MMTs) in 2000 to around 469 MMTs in 2014, and production increased dramatically from 122 MMTs in 2000 to 480 MMTs in 2014 (China Energy Statistics Yearbook, 2015). Various official public sources have reported that China is the world's largest consumer and producer of coke.

Following the rapid increase in the consumption and production of coke, China launched coke spot and future markets in 2009 and 2011, respectively, to facilitate coke trading, inventory management, and, more importantly, risk hedging for domestic coke users and producers. However, due to its immaturity, the coke market is very speculative, thus indicating that the coke price is very sensitive and vulnerable to market shocks. Accordingly, understanding the relationship between the domestic coke price and its related risk factors is particularly important for risk management of the coke price.

According to the Energy Information Administration, 2014, China's coal mines primarily produce bituminous coal and fair amounts of anthracite and lignite. These elements make up steam coal, which is mainly used to generate electricity and produce heat in the industrial sector, and coking coal, which is primarily used to produce coke for iron smelting and steel production. Due to its close relationship with these production processes, the coke industry is reported to be the third largest coal consumer in China after the power generation and manufacturing industries (Huo et al., 2012), and the coal price is a primary determinant of the coke price. At the same time, given that coke is an indispensable raw material in iron and steel production, the production capacities of the iron and steel industries are another main factor in determining the coke price. Moreover, the Coke Manual (2011) published by the Bohai Commodity Exchange (BCE) points out that coke inventories and domestic macro fundamentals should also be considered when analyzing the coke price.

However, with China's integration into the world energy market, it is evident that domestic risk factors cannot provide the complete risk pattern for the coke price. The high correlation between the world oil and
coal prices since 2008 and China’s dominant role in importing international oil and coal has been greatly emphasized in the literature (Yang et al., 2012; Zaklan et al., 2012). These factors have also aroused the attention of domestic coke users and producers on world energy price disturbances. As the benchmark in the world energy market, the crude oil price is generally regarded as more volatile than other energy products, and is considered to be the best candidate for the risk transmission to other markets (Regnier, 2007; Lautier and Raynaud, 2012). The external risk factor of the world oil price is especially highlighted in the Coke Manual. Nonetheless, in sharp contrast to the abundant data analyses on domestic risk factors, thus far, there has been no related analysis of the relationship between China’s coke price and the world oil price.

Furthermore, the ongoing financialization of commodities, the advent of the 2008–2009 global financial crisis, and the subsequent global economic slowdown have been accompanied by extreme movements in oil prices, which have attracted the attention of market participants worldwide. In recent years, the price of oil has fluctuated at levels that have not been observed since the energy crisis of the 1970s. For example, the price of WTI (West Texas Intermediate) crude oil rose from 37 dollars per barrel to a historic maximum of 145 dollars per barrel from the beginning of 2003 to mid-2008, and then decreased sharply to 33 dollars per barrel at the end of 2008. Following a mild upward price trend throughout 2009, the price promptly rose from 80 dollars per barrel to almost 100 dollars per barrel at the end of 2010 and into 2011. Recently, weak economic growth coupled with surging U.S. production and OPEC’s decision to not cut oil production caused the oil price to collapse to around 40 dollars per barrel from about 105 dollars per barrel in April 2014. Notably, significant fluctuations have been observed in other energy markets, such as coal, electricity, natural gas and refined petroleum, along with the dramatic changes in the world oil price. The literature has mostly analyzed such extreme market conditions using uncertain macro fundamentals, fads, and herd behavior, and emphasized that they can impose non-negligible effects on investment decisions and macro policy making (see Ghorbel and Travelsi, 2014; Joëts, 2014; Tong et al., 2013; Yang et al., 2012, among others). This further motivates us to examine the relationship between China’s coke price and the world oil price, especially in light of the extreme movements in the oil price.

In addition to revealing the implications of risk management for coke users and producers, uncovering the relationship between China’s coke price and the world oil price, especially with respect to the extreme oil price fluctuations, is expected to assist domestic policy makers in regulating the market risk, facilitate the development of the domestic energy-related markets, improve the asset pricing of domestic energy products, and help adjust energy policies to reduce China’s heavy reliance on imported oil.

To this end, we use an autoregressive conditional jump intensity (ARJJ) model with the GARCH process to describe the world oil price and China’s coke market, given its speculative characteristics, thus guaranteeing that the jumps in the oil and coke prices will be captured. In particular, we add the world oil price jump intensity and the negative and positive returns of the oil price into the mean equation of China’s coke returns to comprehensively investigate the effect of world oil price shocks on China’s coke price. In addition to focusing on the effect of extreme oil price shocks (oil price jump intensity) on China’s coke price in average conditions, we further investigate how the commodity prices co-move in extreme market cases. Then, using the estimates from the ARJJ-GARCH models, we apply diverse copulas (including the static and time-varying Gaussian copula, Student-t copula, Clayton copula and its rotation, and Gumbel copula and its rotation) to further examine the dependence structure of coke and oil.

The main findings of this paper, which are based on a daily spot sample of China’s coke price and the WTI price from 2009 to 2015, are summarized as follows. First, extreme jumps are evident in both the world oil price and China’s coke price, thus confirming that they are not only characterized by GARCH volatility but also by jump behaviors. Second, negative oil price shocks lead to falls in China’s coke returns on the following day, while the effect of positive oil price shocks is insignificant. China’s current coke returns also positively react to the very recent jump intensity in the world oil price, while the two-day lagged oil price jump intensity has no significant effect. Third, there is time-varying and volatile lower tail dependence between the world oil price and China’s coke price, indicating co-movements in their extreme negative returns.

The remainder of this paper is organized as follows. Section 2 provides a brief overview of China’s coke trading market. Section 3 presents the literature review. The methods, including the ARJJ-GARCH model and copulas, are introduced in Section 4. The descriptive statistics of the data and the empirical results are presented in Sections 5 and 6, respectively. Section 7 concludes the paper and presents the final discussion.

### 2. China’s coke trading market

Spot trading on China’s coke market was launched on the BCE on December 18, 2009 with the purpose of facilitating coke trading, inventory management, and price risk hedging. To maintain continuous spot transactions, the BCE is structured on the basis of a daily delivery declaration and delay delivery compensation system. Similar to futures trading, the continuous spot trading system allows traders to hold short positions and uses the T + 0 transaction mechanics. However, different from futures trading, spot trading requires a higher margin ratio of 20% of the contract value. The tick size and trading unit are set at RMB 2/MT and 1 MT, respectively. The daily price limit is +/− 8% of the guided price on the first listing day, whereas after that date it is +/− 8% of the last settlement price. Coke spot trading is based on the physical delivery and the trading hours are divided into three sessions: 9:00–3:00, 9:00–11:30, and 13:30–16:00 (all Beijing time).1

To better hedge the risk of the coke price, a coke futures market was subsequently launched on August 15, 2011 on the Dalian Commodity Exchange (DCE). Coke futures trading requires a minimum margin ratio of 5% of the contract value. The tick size and trading unit are RMB 0.5/MT and 100MT/contract, respectively. The daily price limit is set to be 4% of the last settlement price. The expiration day of a coke futures contract is the second day after the last trading day of the delivery month (coke contracts are monthly contracts, comprising of 12 contracts per year, and the last trading day is the tenth trading day of the delivery month) and is based on physical delivery. The trading hours for coke futures are divided into two sessions: 9:00–11:30 and 13:30–15:00 from Monday to Friday (all Beijing time).2

Thus far, three coal-related spots (coke, coking coal, and steam coal) have been introduced on the BCE. Coke spot trading was introduced first and has the largest trading volumes, with the daily trading volume being around 153 thousand MT on average. Coke, coking coal, and steam coal futures are also traded on the DEC. Like the coke spots, coke futures were launched earlier than the other two coal-related futures, and are the most actively traded of the three coal-related futures, with an average daily trading volume around 399 thousand contracts. Compared with coke users and producers, speculators and arbitragers comprise a larger proportion of the participants in the domestic coke spot and futures markets. Moreover, according to the averaged ratio of the trading volume and open interest, the coke spot and futures markets are mainly characterized by speculation.

### 3. Literature review

As noted above, no studies have empirically examined the relationship between China’s coke price and the world oil price. The most

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1 More details can be found at [www.bce.com.cn](http://www.bce.com.cn).
2 More details can be found at [www.dce.com.cn](http://www.dce.com.cn).
relevant studies in this area are concerned with the relationship between the oil price and the prices of other energy commodities, such as coal, natural gas, consumer liquid fuel (e.g., diesel, petrol, and heating oil), and electricity. Among them, the early studies examine the long-run relationship between the oil price and other energy prices (e.g., Serletis, 1994; Serletis and Kemp, 1998; Serletis and Rangel-Ruiz, 2004), while the more recent studies focus more on the short-run links that may lead energy prices to diverge from the long-term equilibrium (e.g., Honarvar, 2009; Lescaroux, 2009; Moutinho et al., 2011; Sensory et al., 2015; Zavala, et al., 2015).

In addition to the abovementioned studies, the extreme movements in energy prices (mainly the oil price) have attracted significant research attention in recent years. Following Chan and Maheu’s (2002) suggestion that the popular GARCH type model is designed to capture smooth persistent changes only and is not suited to explain the large and sudden changes found in asset returns, a number of subsequent studies began to use their ARJI model to capture both the volatility clustering and changes in the intensity of extreme movements of energy prices. For example, Lee et al. (2010) used a component-ARJI model with structural break analysis to examine WTI crude oil spot and future price behaviors, and identified the existence of permanent and transitory components in the conditional variance. Gronwald (2012) used a combined jump GARCH model to investigate the behavior of daily, weekly, and monthly oil prices, and verified that oil prices are not only characterized by GARCH but also by the conditional jump behavior, and showed that a considerable portion of the total variance is triggered by sudden extreme price movements. Wilmot and Mason (2013) allowed the potential presence of jumps in the spot and future prices of crude oil, and found that this consideration can improve the model's ability to explain the dynamics of crude oil prices.

However, recently, Wang and Zhang (2014) argued that the abovementioned studies only identify the existence of oil price jumps and pay little attention to how the jump behaviors in the crude oil prices affect other markets. To this end, Wang and Zhang (2014) used the ARJI-GARCH model to examine how jumps in the crude oil market affect China’s bulk commodity prices. Although Wang and Zhang (2014) represent an important step in modeling the effect of jump behavior in the crude oil price on other commodity prices, the issue of how the oil price jumps affect other energy commodity prices remains unclear.

Nonetheless, in line with the increasing attention being paid to the extreme risks in energy prices, an emerging stream of literature has been focusing on the co-movements between the oil price and other energy prices under extreme market circumstances. For instance, Joets (2014) showed that energy (oil, gas, coal, and electricity) price co-movements increase during extreme fluctuations and that this tendency appears to be stronger during bear markets. Using weekly data on WTI crude oil, heating oil, gasoline, and natural gas prices, Koch (2014) suggested that tail events across energy markets cannot only be explained by the real demand fundamentals, but also by the changes in the net long positions of hedge funds and the TED spread. Tong et al. (2013) found that crude oil and refined petroleum prices tend to move together during market upturns and downturns, while Aloui et al. (2014) reported that crude oil and natural gas prices tend to co-move closely in bullish periods but not during bearish periods. After examining the tail distribution patterns and tail dependence of the price returns of WTI oil, natural gas, and heating oil prices, Ghorbel and Travelsi (2014) further provided acceptable VaR (value-at-risk) estimates of energy portfolios for investors. From this perspective, in addition to quantifying the effects of extreme movements in one energy price (mainly the oil price) on other energy commodity returns in normal time, it is necessary to model their extreme co-movements and to examine the extreme risk of energy prices.

In terms of modeling the extreme co-movements of asset returns, copula functions have proven to be a very advantageous approach. Specifically, without using discretion to define extreme observations, copula functions can exhibit diverse patterns of market tail dependence. Moreover, without assuming multivariate normality while based on the marginal distributions, copulas have great suitability and flexibility in building the joint distribution of asset returns (Wen et al., 2012). Among the above studies, Tong et al. (2013); Aloui et al. (2014), and Ghorbel and Travelsi (2014) used copulas to examine the extreme co-movements between the oil price and other energy commodity prices. Overall, an increasing number of studies have applied copulas to study energy markets in recent years (see Aloui et al., 2013a; Aloui et al., 2013b; Chang, 2012; Reboreda, 2011, 2013, 2015; Wen et al., 2012; among others).

This paper extends the existing research in the following two dimensions. First, we attempt to fill the current research gap by investigating the relationship between the world crude oil price and China’s coke price. Second, because the effect of crude oil price jumps on the returns of other energy commodity prices in normal time remains under-researched, and extreme co-movements between energy prices is another necessary dimension of the extreme risk of energy prices, we combine the ARJI-GARCH model and copula functions to measure the relationship between China’s coke price and the world oil price. To the best of our knowledge, few recent studies have used this approach in energy market contexts. The only exception is Chang (2012), who investigated the dependence between the crude oil spot and futures markets using the mixed copula-based ARJI-GARCH model. Hence, following Chang (2012), we use more diverse copula functions, namely, static and dynamic copulas of the Gaussian, Student-t, Clayton and its rotation, and Gumbel and its rotation, which are expected to more fully describe the relationship between China’s coke price and extreme changes in the world crude oil price.

4. Methods

4.1. ARJI-GARCH model for the world oil price and China’s coke price

The ARJI model proposed by Chan and Maheu (2002) allows us to simultaneously consider the persistence in the conditional variance and jump behavior in asset prices. The ARJI model combined with a GARCH process for the world oil price is given as follows:

\[
R_{oil,t} = \mu_{oil} + \sum_{i=1}^{l} \phi_{oil,i} R_{oil,t-i} + \sqrt{h_{oil,t}} \epsilon_{oil,t} + \sum_{k=1}^{m} Y_{oil,t,k}.
\]

\[
z_{oil,t} \sim NID(0,1), Y_{oil,t,k} \sim N\left(\theta_{oil,k}, \delta_{oil,k}^2\right).
\]

Although the ARJI model has been widely used in traditional financial markets (e.g., Chen and Shen, 2004; Chiou and Lee, 2009; Daal et al., 2007), its application to energy prices is still emerging.
where Eq. (1) is the conditional mean equation of the world oil return \( R_{oil,t} \), \( \mu_{oil} \) is the constant term, and \( \phi_{oil,i} \) \((i = 1, 2, \ldots, l)\) is the coefficient of the AR process. To complete the conditional volatility dynamics for returns, \( h_{oil,t} \) follows a GARCH \((p, q)\) process (Bollerslev, 1986), which is given by:

\[
h_{oil,t} = \omega_{oil} + \sum_{i=1}^{q} \alpha_{oil,i} \epsilon_{oil,t-i}^2 + \sum_{j=1}^{p} \beta_{oil,j} h_{oil,t-j},
\]

where, following Chan and Maheu (2002), \( \epsilon_{oil,t} = R_{oil,t} - \mu_{oil} - \sum_{j=1}^{l} \phi_{oil,j} \epsilon_{oil,t-j} \). The specification of \( \epsilon_{oil,t} \) contains the expected jump component, thus allowing it to propagate and affect future volatility through the GARCH variance factor.

In Eq. (1), the conditional jump size \( Y_{oil,t} \) is assumed to be normally distributed with a mean \( \theta_{oil} \) and variance \( \xi_{oil} \) given the history of returns \( R_{oil,t} = \{ R_{oil,t-1}, R_{oil,t-2}, \ldots, R_{oil,1} \} \). \( n_t \) is the discrete counting process governing the number of jumps that arrive between time \( t-1 \) and \( t \) in the world oil price, which follows a Poisson distribution with the parameter \( \lambda_{oil,t} > 0 \) and density:

\[
P(n_t = j \mid t_{-1}) = \frac{\exp(-\lambda_{oil,t}) \lambda_{oil,t}^j}{j!} \quad j = 0, 1, 2, \ldots
\]

where \( \lambda_{oil,t} \) is the jump intensity and implies the conditional expectation of the counting process under \( l_{-1} \), \( \lambda_{oil,t} \) is assumed to follow:

\[
\lambda_{oil,t} = \lambda_{oil,0} + \rho_{oil,1} \lambda_{oil,t-1} + \rho_{oil,2} \xi_{oil,t-1},
\]

where \( \lambda_{oil,0} > 0, \rho_{oil,1} > 0, \rho_{oil,2} \geq 0; \xi_{oil,t-1} \) is the jump intensity residual, which is calculated as:

\[
\xi_{oil,t-1} = E[n_t \mid t_{-1}] - \lambda_{oil,t-1} = \sum_{j=0}^{\infty} j P(n_t = j \mid t_{-1}) - \lambda_{oil,t-1}.
\]

Having observed \( R_{oil,t} \) and using the Bayes rule, the ex-post probability of the occurrence of \( j \) jumps at time \( t \) can be inferred and is defined as:

\[
P(n_t = j \mid t) = \frac{f(R_{oil,t} \mid n_t = j, t_{-1}) P(n_t = j \mid t_{-1})}{P(R_{oil,t} \mid t_{-1})}, \quad j = 0, 1, 2, \ldots
\]

and the log likelihood function of the ARJ-GARCH model for the world oil price can be written as follows:

\[
L(\phi_t) = \sum_{t=1}^{T} \ln \left[ P(R_{oil,t} \mid n_t = j, \phi_t) \right],
\]

\[
P(R_{oil,t} \mid t_{-1}) = \sum_{j=0}^{\infty} f(R_{oil,t} \mid n_t = j, t_{-1}) P(n_t = j \mid t_{-1}), \quad j = 0, 1, 2, \ldots
\]

where \( T \) is the sample size and \( \phi_t \) represents all of the parameters to be estimated.

As for China’s coke price, we still use the ARJ-GARCH model to describe the conditional variance and jump behavior in prices. To fully examine the potential effect of the world oil price, the conditional mean equation of China’s coke price is given by:

\[
R_{coke,t} = \mu_{coke} + \sum_{i=1}^{l} \phi_{coke,i} R_{coke,t-i} + \sum_{i=1}^{m} k_{1i} P_{R_{oil,t-i}} + \sum_{i=1}^{w} k_{2i} N_{R_{oil,t-i}} + \sum_{i=1}^{s} k_{3i} R_{coke,t-i} + \sum_{i=1}^{25} d_{i} \lambda_{coke,t-i} + \sqrt{h_{coke,t}} \epsilon_{coke,t} + \sum_{k=1}^{m} Y_{coke,t,k},
\]

where \( P_{R_{oil,t-i}} = \max(R_{oil,t-i}, 0) \) is defined as a positive oil price shock while \( N_{R_{oil,t-i}} = \min(R_{oil,t-i}, 0) \) is a negative oil price shock. \( \lambda_{coke,t} \) is the jump intensity of the oil price, which allows us to determine how China’s coke returns are affected by the jump behavior in the world oil price.

The \( \epsilon_{coke,t} \) still follows a GARCH \((p, q)\) process as in Eq. (3), with the parameters of \( \phi_{coke}, \alpha_{coke}, \beta_{coke} \). However, the specification of \( \epsilon_{coke,t} \) is presented as:

\[
\epsilon_{coke,t} = R_{coke,t} - \mu_{coke} - \sum_{i=1}^{l} \phi_{coke,i} R_{coke,t-i} - \sum_{i=1}^{m} k_{1i} P_{R_{oil,t-i}} - \sum_{i=1}^{w} k_{2i} N_{R_{oil,t-i}} - \sum_{i=1}^{s} k_{3i} R_{coke,t-i} - \sum_{i=1}^{25} d_{i} \lambda_{coke,t-i}.
\]

As with \( Y_{oil,t} \), the conditional jump size \( Y_{coke,t,k} \) of Eq. (10) is assumed to be normally distributed with a mean \( \theta_{coke} \) and variance \( \xi_{coke} \) given the history of returns \( R_{coke,t} \) \( R_{coke,t-1}, R_{coke,t-2}, \ldots, R_{coke,1} \). \( n_t \) is the number of jumps arriving between time \( t-1 \) and \( t \) in the coke price, which follows a Poisson distribution with the parameter \( \lambda_{coke,t} > 0 \) and the density is in the same form as Eq. (4). The jump intensity of the coke price \( \lambda_{coke,t} \) follows the ARMA\((1, 1)\) process as in Eqs. (5) and (6), although with the parameters \( \lambda_{coke,0} > 0, \rho_{coke,1} > 0, \rho_{coke,2} \geq 0, \) and the jump intensity residual \( \xi_{coke,t-1} \).

With the observed \( R_{coke,t} \) and the Bayes rule, the ex-post probability of the occurrence of \( j \) jumps at time \( t \) can also be inferred and is given by a formula similar to Eq. (7). The log-likelihood function of the ARJ-GARCH model for the coke price can be written as follows:

\[
L(\phi_t) = \sum_{t=1}^{T} \ln \left[ P(R_{coke,t} \mid n_t = j, \phi_t) \right],
\]

For the coke price, \( R_{coke,t} \) needs to be substituted with \( R_{coke,t} \) in Eq. (7).
\[ P(R_{\text{coke}}|l_{t-1}) = \sum_{j=0}^{m} f(R_{\text{coke}}|l_t = j, l_{t-1}) P(n_t = j|l_{t-1}), \quad j = 0, 1, 2, \ldots \]  

where \( T \) is the sample size and \( \Phi \) represents all of the parameters to be estimated.

Finally, it should be noted that to stress the important effect of the oil price, especially the effect of the oil price jump intensity, we also estimate a number of basic models for the Chinese coke price in the empirical analysis section, including the ARJI-GARCH model without any oil price effect, ARJI-GARCH model with the effect of lagged oil returns, and ARJI-GARCH model with the asymmetric effect of lagged oil returns.

4.2. Copulas

In Section 4.1, we showed that the effect of the jump behavior in the oil price (i.e., the intensity of extreme oil price movements) on coke returns in average conditions can be examined based on Eq. (10). However, to further investigate how the two commodities co-move in extreme cases, we use copula functions.

According to Sklar’s theorem, copula functions are a very convenient tool for building a multivariate distribution for assets with any choice of marginal distributions for each individual asset. Concretely, a two-dimensional joint distribution function \( G \) with continuous marginal distributions \( F_x \) and \( F_y \) can be given by \( G(x, y) = C(F_X(x), F_Y(y)) \). Thus, the joint distribution function is given by the marginal distributions (the cumulative distributions for each individual asset) and the dependence structure is described by a copula function. As a density function, the above function can be written as \( g(x, y) = f_x(x) f_y(y) \cdot c(F_X(x), F_Y(y)) \), with \( f \) being the probability density function of the marginal distribution of the asset price and \( c \) being the copula density.

Based on Chan and Maheu (2002), the adequacy of the ARJI-GARCH model should not only be examined by investigating whether there is serial correlation in the standardized residuals and the jump intensity residuals, but also by checking whether the cumulative function of the individual asset returns is uniform \((0, 1)\). The cumulative function of individual assets are required for copula functions, and whether they are uniform \((0, 1)\) needs to be verified before introducing the copula functions. Under the ARJI-GARCH model, we can obtain the marginal distributions for the world oil price and China’s coke price, \( F_{\text{oil}}(R_{\text{oil}}) \) and \( F_{\text{coke}}(R_{\text{coke}}) \). Setting \( F_{\text{oil}}(R_{\text{oil}}) = u_t \) and \( F_{\text{coke}}(R_{\text{coke}}) = v_t \), the distributions are obtained as follows:

\[
\begin{align*}
    u_t &= F_{\text{oil}}(R_{\text{oil}}|l_{t-1}) \\
    &= \sum_{j=0}^{m} P(n_t = j|l_{t-1}) \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp \left[ -\frac{1}{2} \left( \frac{h_{\text{oil}} + \imath j^2 c_{\text{oil}}}{} \right) \right] \\
    v_t &= F_{\text{coke}}(R_{\text{coke}}|l_{t-1}) \\
    &= \sum_{j=0}^{m} P(n_t = j|l_{t-1}) \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp \left[ -\frac{1}{2} \left( \frac{h_{\text{coke}} + \imath j^2 c_{\text{coke}}}{} \right) \right]
\end{align*}
\]  

We perform the Kolmogorov–Smirnov (K-S) test and Anderson–Darling (A-D) test to examine whether \( u_t \) and \( v_t \) from ARJI-GARCH are uniform \((0, 1)\). These tests are discussed in the empirical section. Then, \( G(R_{\text{oil}}, R_{\text{coke}}) = C(u_t, v_t) \) and its density function is \( g(x, y) = P(R_{\text{oil}}) \cdot P(R_{\text{coke}}) \cdot c(u_t, v_t) \).

In addition to being able to build more effective multivariate distributions, copula functions exhibit diverse patterns of tail dependence between markets, which is another obvious advantage and the main purpose of using them in this analysis.

The lower and upper tail dependence between markets are respectively defined as follows:

\[
\begin{align*}
    \lambda_l(v) &= \lim_{v \to 0} p[X \leq F_X^{-1}(v)/Y \leq F_Y^{-1}(v)] = \lim_{v \to 0} \frac{C(v, v)}{v}, \quad (16) \\
    \lambda_u(v) &= \lim_{v \to 1} p[X \leq F_X^{-1}(v)/Y \leq F_Y^{-1}(v)] = \lim_{v \to 1} \frac{1 - 2v + C(v, v)}{1 - v}. \quad (17)
\end{align*}
\]

Eqs (16) and (17) give the probability that an event with probability lower than \( v \) occurs in \( X \) given that an event has occurred with probability lower than \( v \) in \( Y \). \( C(.) \) is the cumulative distribution function of the copula. If \( \lambda_l \) is close to one, it indicates that both returns have extreme negative values and the degree of co-movement in negative extremes is large, whereas if \( \lambda_u \) is close to one, it implies that the degree of co-movement in positive extremes is large.

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As suggested by Patton (2006), if a misspecified model is used for the marginal distributions, then the probability integral transforms will not be uniform \((0, 1)\) and any copula model will automatically be misspecified.
The daily trading data for China's coke spot market begins from December 18, 2009. As the trading data for coke futures are still very limited, we do not use these future data in this paper.

Fig. 1. Returns for the world oil price and China's coke price.

In this paper, we use several popular copula functions that emphasize tail dependence, including the Student- $t$ copula, the Clayton copula and its rotation, and the Gumbel copula and its rotation. The Gaussian copula, which is the benchmark copula in economics, is also considered. Considering that the tail dependence is potentially time-varying, the parameters in these copula functions follow the process of ARMA(1, 10) as proposed in Patton (2006):

$$W_t = A \left( \psi_0 + \psi_1 W_{t-1} + \psi_2 \sum_{i=1}^{10} [u_{t-i} - v_{t-i}] \right).$$  \hspace{1cm} (18)

where $W_t$ stands for the related parameter of a copula function. Additional details of the copula functions used in this paper and their evolution are described in Appendix A.

We first obtain the parameters of the marginal distribution (ARJI-GARCH) of each asset return. Then, based on the two-stage estimation procedure proposed by Joe (1997), in the second stage, the parameters of the copulas are obtained by solving the following problem:

$$\hat{c} = \arg \max_{\phi} \sum_{t=1}^{T} \ln c(\hat{u}_t, \hat{v}_t; \phi).$$  \hspace{1cm} (19)

where $\phi$ are the copula parameters.

5. Data

We use the spot price of coke traded on the BCE and use the WTI spot oil price to represent the world oil price, as the WTI crude oil price is one of the benchmarks in the world oil market. To measure the relationship between the world oil price and China's coke price more accurately, the exchange rates are used to convert the nominal dollar price of oil to the Chinese yuan price. Due to data availability, the daily sample covers December 18, 2009 to April 30, 2015. China's coke price data are obtained from the trading software of the BCE. The WTI spot data are from the IEA and the exchange rate data are from the Federal Reserve Bank of St. Louis. The asset returns are calculated as 100 times the difference in the log of prices.

Fig. 1 presents the daily returns of each asset. Clusters of significant WTI return volatilities can be found from 2010 to the middle of 2012 and then much greater volatilities are observed from the end of 2014 to the end of the sample. For China's coke price, the return volatilities were relatively low between 2011 and mid-2012, while greater volatilities emerged during the first half of 2010 and from mid-2012 to the beginning of 2014. In sharp contrast with the large volatilities of WTI returns from the end of 2014 to the end of the sample, China's coke returns show little variation during this time.

Table 1 provides the descriptive statistics. The sample mean of China's coke returns is smaller than that of WTI returns while the WTI return variations are larger than those of China's coke returns. Moreover, all of the asset returns are left skewed and have excess kurtosis, implying that the probability of extreme negative price changes is larger than that of extreme positive changes, and together with the Jarque–Bera test, suggesting a non-normal distribution for asset returns. Finally, the Ljung-Box statistics suggest the presence of serial correlation in (squared) returns.

<table>
<thead>
<tr>
<th></th>
<th>WTI</th>
<th>Coke</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>−0.024</td>
<td>−0.048</td>
</tr>
<tr>
<td>Std.</td>
<td>1.916</td>
<td>1.297</td>
</tr>
<tr>
<td>Skewness</td>
<td>−0.183 ***</td>
<td>−0.685 ***</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.528 ***</td>
<td>6.598 ***</td>
</tr>
<tr>
<td>J-B</td>
<td>673.475 ***</td>
<td>2429.750 ***</td>
</tr>
<tr>
<td>Q(15)</td>
<td>18.034</td>
<td>37.409 ***</td>
</tr>
<tr>
<td>Q^2(15)</td>
<td>337.355 ***</td>
<td>74.778 ***</td>
</tr>
</tbody>
</table>

Notes. Daily observations are for the period of Dec. 21, 2009 to Apr. 30, 2015. The Jarque–Bera (J-B) statistic tests for the null hypothesis of normality in the sample returns distribution. The $Q(15)$ is the Ljung-Box Q test of serial correlation of up to 15 lags in the returns. ***, **, * indicate statistical significance at the 1%, 5% and 10% level respectively.
### 6. Empirical results

#### 6.1. Estimates of the ARJI-GARCH model

Table 2 shows the estimates of the ARJI-GARCH models for returns of the world oil price and China’s coke price. Following Chan and Maheu (2002) and Chang (2012), the number of jumps is set to 20 for all of the models. The number of lags in the conditional mean equation is selected based on the BIC (Bayesian information criterion), which is known to lead to a parsimonious specification (Beine and Laurent, 2003). Following the literature (e.g., Chan and Maheu, 2002; Chang, 2012; Wang and Zhang, 2014; and Wang and Zhang, 2014), orders $p$ and $q$ of the GARCH process are both specified as $1$.\(^7\)

As the misspecification tests show, the null hypothesis of no serial correlation in the squared standardized residuals (jump intensity residuals) cannot be rejected in all models. In addition, the $p$-values for the K-S test and A-D test indicate that the probability integral transforms are uniform $(0, 1)$. Hence, the ARJI-GARCH model for the world oil and China’s coke returns adequately describes their marginal distributions and the relevant information at time $t$ but also on the volatility at time $t-1$ but also on the relevant information at the same time.

The significant effect of the world oil price on China’s coke price and the highest log-likelihood value are written in bold.

The estimates of the ARJI-GARCH model for WTI returns are shown first. In the conditional mean equation, the coefficient $\delta_{\text{out}}$ is negative and statistically significant, indicating that WTI returns are negatively related to one lag of their own returns. In the conditional volatility equation, the coefficients $\phi_{\text{out}}$ and $\varphi_{\text{out}}$ are all significant at the 1% level, implying that the volatility of WTI crude oil returns at time $t$ not only depends on the volatility at time $t-1$ but also on the relevant information at the same time. As for the jump behavior in world oil prices, the significant mean $\theta_{\text{out}}$ and variance $\xi_{\text{out}}$ of the jump size imply that sudden extreme movements occur with the abnormal news flowing into the world oil market. The significance of coefficients $\lambda_{\text{out},0}$ and $\rho_{\text{out},2}$ indicates that the ARJI model is appropriate to describe the jump behavior in the WTI oil prices and the values of $\rho_{\text{out},1}$ and $\rho_{\text{out},2}$ imply that the current jump intensity $\lambda_{\text{out}}$ is almost equally affected by the most recent jump intensity $\lambda_{\text{out},-1}$ and intensity residuals ($\xi_{\text{out},-1}$).

#### 6.1.1. The effect of the world oil price on China’s coke price

To emphasize the role of the oil price, especially that of the oil price jump intensity, in describing the dynamics of coke returns, some basic models are estimated in Table 2, which are denoted as Coke (A), Coke (B), and Coke (C), respectively.

The column denoted Coke (A) of Table 2 shows the estimates of the ARJI-GARCH model without any oil price effect. The volatility pattern of China’s coke returns is found to be similar to that of world oil returns, with the coke return volatility at time $t$ depending on both the volatility and the relevant information at time $t-1$. The significant variance $\delta_{\text{coke}}$ of the jump size implies that sudden extreme jumps are also evident in China’s coke price, and the significant $\rho_{\text{coke},1}$ and $\rho_{\text{coke},2}$ indicate that the current jump intensity in China’s coke price ($\lambda_{\text{coke}}$) is related to the lagged coke jump intensity and past shocks.

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\(^7\) According to Brooks (2008), because a GARCH family model with 1 lag order can sufficiently capture the volatility clustering in assets returns, few financial studies have considered or used the high order model.
The effect of lagged oil returns is then considered in the ARJI-GARCH model for China’s coke returns. The columns denoted Coke (B) and Coke (C) show the results of the models with the symmetric and asymmetric effects of lagged oil returns, respectively. The significance of coefficient \( k \) in Coke (B) indicates that the current returns of China’s coke price positively respond to the lagged WTI oil returns. Furthermore, in Coke (C), the significance of coefficients \( k_{22} \) and \( k_{11} \) implies that the current returns of China’s coke price are positively affected by the negative lagged WTI oil returns, and that they are insignificantly affected by the positive lagged oil returns.

Finally, the column denoted by Coke provides estimates from the ARJI-GARCH model for China’s coke returns that considers both the asymmetric effect of the lagged oil returns and the effect of the lagged oil price jump intensity. The asymmetric effect of the lagged WTI oil returns remains in this model. As for the effect of the oil price jump intensity, the significant and positive \( d_1 \) means that the one-day lag of jump intensity of the WTI oil price \( (\lambda_{WTI}) \) positively affects China’s current coke returns, while the insignificant \( d_2 \) implies that the two-day lag of jump intensity of the WTI oil price does not affect the coke returns.

These results indicate that China’s coke price returns only react to the very recent jump intensities of the WTI oil price. This may be attributable to the speculative Chinese coke market, which is inclined to overreact to very recent oil price shocks. Comparing the values of \( k_{22} \) and \( d_1 \), the effect of the oil price jump intensity is confirmed as being obviously stronger than that of the lagged negative oil price returns.

The coefficients in the conditional volatility equation of China’s coke price across the column Coke (A) until the last column are all very similar. Given the coefficients of \( \alpha_{oil} \), \( \beta_{oil} \), and \( \alpha_{coke} \), we find that the volatility clustering in the coke price is weaker than that in the crude oil price, and the coefficient values of \( d \) show that the variance in the jump size in the world oil market is at least twice as large as that in China’s coke market.

In summary, all of the models in Coke (A), (B), (C), and Coke in Table 2 are adequate for describing the marginal distribution of China’s coke returns (based on the Ljung-Box statistics for squared standardized residuals and jump intensity residuals, and on the \( p \)-values of the K-S test and A-D test). However, given the significance of the asymmetric effect of the lagged oil price returns and the effect of the lagged oil price jump intensity, together with their log-likelihoods, the ARJI-GARCH model presented in the last column is confirmed as being the optimal model. Hence, the oil price effect needs to be fully considered when modeling China’s coke price returns.

6.1.2. Volatility patterns of the world oil price and China’s coke price

Based on the estimates from the ARJI-GARCH model presented in the WTI and Coke columns in Table 2, Fig. 2 displays the jump intensities of...
The best fit static copula and time-varying copula are noted in bold.

### 6.2.2. Time paths of tail dependence

Fig. 4 depicts the development of the lower tail dependence between the world oil price and China’s coke price according to the two most optimal copula functions, namely, the time-varying rotated Gumbel copula and the static Clayton copula. The figure shows that the lower tail dependence obtained from the static Clayton copula is very close to zero whereas, in contrast, the lower tail dependence from the time-varying rotated Gumbel copula is much higher and more volatile. Because the time-varying rotated Gumbel copula is more optimal than the static Clayton copula, estimates from the static Clayton copula would under-estimate the extreme downside risk between these two markets.

### 7. Conclusion and discussion

In this paper, using a daily sample of China’s coke spot prices and the WTI spot prices from December 18, 2009 to April 30, 2015, and the

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**Table 3**

Estimates of static and time-varying copulas.

<table>
<thead>
<tr>
<th>Static normal copula</th>
<th>TV normal copula</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ</td>
<td>ψ₀</td>
</tr>
<tr>
<td>0.057***</td>
<td>0.135*</td>
</tr>
<tr>
<td>(0.029)</td>
<td>(0.077)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Static Student-ₜ copula</th>
<th>TV Student-ₜ copula</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ</td>
<td>ψ₀</td>
</tr>
<tr>
<td>0.061*</td>
<td>0.094</td>
</tr>
<tr>
<td>(0.030)</td>
<td>(0.096)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Static rotated Clayton copula</th>
<th>TV rotated Clayton copula</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ</td>
<td>ψ₀</td>
</tr>
<tr>
<td>0.046***</td>
<td>-2.984***</td>
</tr>
<tr>
<td>(0.031)</td>
<td>(1.361)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Static Gumbel copula</th>
<th>TV Gumbel copula</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ</td>
<td>ψ₀</td>
</tr>
<tr>
<td>1.100***</td>
<td>-1.166</td>
</tr>
<tr>
<td>(0.058)</td>
<td>(1.908)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Static rotated Gumbel copula</th>
<th>TV rotated Gumbel copula</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ</td>
<td>ψ₀</td>
</tr>
<tr>
<td>1.100***</td>
<td>-1.519</td>
</tr>
<tr>
<td>(0.035)</td>
<td>(1.587)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ψ</th>
<th>Log-likelihood</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.044</td>
<td>2.984</td>
<td>1.091</td>
</tr>
<tr>
<td>(0.032)</td>
<td>(1.361)</td>
<td></td>
</tr>
</tbody>
</table>

Notes. The table shows the likelihood estimates of static and time-varying (TV) copulas for WTI oil price and China’s coke price. The standard error values are presented in the brackets; Akaike Information Criterion (AIC) values adjusted for small-sample bias are provided for the copula models. The best copula fit is selected based on the minimum AIC value and the maximum Log-likelihood value. *,**,*** indicate statistical significance at the 1%, 5% and 10% level respectively. The best-fit static copula and time-varying copula are noted in bold.
relatively novel ARJI-GARCH-copula models, we find that the world oil price and China’s coke price are characterized by both GARCH volatility and jump behaviors. Moreover, our results show that negative (positive) oil price shocks can lead to falls (have no effect) in China’s coke returns on the following day, the lagged jump intensity in the world oil price can significantly increase China’s current coke returns, and co-movements of the extreme negative returns are time-varying and very volatile. These results are worthy of further discussion.

A number of potential implications for the risk management of coke producers and users can be obtained from the empirical results. (1) Lagged negative oil price shocks lead to falls in the current coke returns, while lagged positive oil price shocks have no effect on today’s coke returns, implying that coke producers should pay more attention to negative news about the world oil price. Moreover, the existence of lower tail dependence indicates that hedging the extreme downside risk of the world oil price is particularly important. (2) The lagged oil price jump intensity can significantly drive up the current coke returns in average conditions, implying that coke users should pay special attention to the jump behavior in oil prices to avoid high use-costs. (3) However, it should also be noted that although China’s coke price is found to react to the lagged oil price jump intensity, this response appears to be very short-lived. Specifically, the coke price only reacts to very recent oil price jump intensity, while the two-day lag of jump intensity of the WTI oil price has no effect on coke prices. Meanwhile, the lower tail dependence between the world oil price and China’s coke price stays below 0.1 in most cases, even though it is time-varying and volatile. This indicates that the diversification potential of world oil futures should be considered when hedging coke price risk. (4) Because jump behaviors are observed in both the world oil price and China’s coke price, using models that only capture the smooth GARCH volatility may underestimate the risk. Similarly, given the volatile lower tail dependence between the world oil price and China’s coke price, static models and models that assume symmetric tail dependence may also underestimate the risk, leading to reduced hedging efficiency.

Our results also have important implications for policy makers. (1) According to the above discussion, the world oil price is identified as a significant risk factor for domestic coke returns. Thus, disturbances in the world oil price should be considered when pricing domestic coke derivatives. (2) Hedging the world oil price risk is necessary for coke users and producers, which implies that crude oil futures need to be introduced along with more energy-related derivatives. However, there are very few energy-related futures in China at present, as the launch of the crude oil futures market has been suspended and the coal-related futures markets are still emerging. Among the emerging coal-related futures markets, the coke futures market has high trading volumes but remains very speculative. Accordingly, policy makers need to consider regulating the speculation risk and maintaining adequate market liquidity. In fact, gradually lowering the dependence on oil imports by diversifying China’s energy consumption structure is an alternative option to mitigate the oil price risk in the domestic coke price. (3) The short-lived response of China’s coke price to the oil price jump intensity and their weak lower tail dependence in most cases motivate us to conclude that domestic factors may play a more dominant role in affecting the coke price. In other words, despite China’s large energy exports and imports, factors such as the individual characteristics of energy products and industries, and domestic macroeconomic fundamentals may largely weaken the relationship between the world oil price and China’s coke price. In practical terms, because coke is produced from coking coal, and then used for iron and steel production or other forms of nonferrous metallurgy, the energy substitution between coke and oil is not direct, whereas the demand in the iron and steel industries and the coking coal price are directly linked with the coke price. Moreover, macroeconomic fundamentals play an important role in driving (dampening) the demand for coke in the iron and steel industries, with rapid (slow) economic growth, easing (tightening) monetary policy, and a relatively low (high) inflation rate stimulating (reducing) demand. In addition, rail capacity, the lifecycles of the iron and steel industries (e.g., construction, machinery, cars, ships, and railways), and industry regulation policies are also likely to affect the coke price.

The Twelfth Five-Year Plan for Chinese Coal Industry Development states that coal-based chemical industries should be further promoted and, according to Yang et al. (2012), the substitution between coal/coal-based chemical products and oil is going to strengthen. Given China’s ever increasing oil imports, domestic coal and coal-related products are predicted to be more vulnerable to the world oil price. Hence, the risk factor of the world oil price is worthy of ongoing research attention. Furthermore, given the speculative nature of the Chinese coke market, a possible extension of our paper would be to examine how coke trading activities affect the price nexus of the world oil and domestic coke markets.

Appendix A. Copula Functions

A.1. Gaussian copula

The Gaussian copula is considered to be the benchmark copula in economics, and is defined by:

$$C_G(u_t, v_t; \rho) = \Phi \left( \Phi^{-1}(u_t), \Phi^{-1}(v_t) \right)$$  

(A.1)
where \( \Phi \) is the bivariate standard normal CDF with correlation \( \rho \) \((-1 < \rho < 1) \), and \( \Phi^{-1}(u_t) \) and \( \Phi^{-1}(v_t) \) are standard normal quantile functions. The Gaussian copula features symmetric tail dependence, while \( \lambda_0 = \lambda_\gamma = 0 \).

A.2. Student-\( t \) copula

The Student-\( t \) copula is another kind of elliptical copula that is often used for the dependence structure. Its equation is given by:

\[
C_t(u_t, v_t; \nu, \rho) = T\left(t_{\nu,1}^{-1}(u_t), t_{\nu,1}^{-1}(v_t)\right)
\]

where \( T \) is the bivariate Student-\( t \) CDF, with a degree-of-freedom parameter \( \nu \) and correlation \( \rho \) \((-1 < \rho < 1) \), and \( t_{\nu,1}^{-1}(u_t) \) and \( t_{\nu,1}^{-1}(v_t) \) are the quantile functions of the univariate Student-\( t \) distribution, with \( \nu \) as the degree of freedom parameter.

The Student-\( t \) copula also features symmetric tail dependence, while \( \lambda_0 = \lambda_\gamma = 2t_{\nu,1}^{-1}\left(-\sqrt{\nu} + 1\sqrt{1-\rho}/\sqrt{1+\rho}\right) > 0 \), where \( t_{\nu,1}(\gamma) \) is the CDF of the Student-\( t \) distribution with degree of freedom \( \nu + 1 \), and \( \rho \) is the linear correlation coefficient.

A.3. Clayton copula and its rotation

The Clayton copula is good at characterizing the asymptotic lower tail dependence:

\[
C_C(u_t, v_t; \gamma) = \left(\max(u_t^{-\gamma} + v_t^{-\gamma} - 1; 0)\right)^{-1/\gamma}
\]

while its rotation can well consider the upper tail dependence:

\[
C_{CC}(u_t, v_t; \gamma) = u_t + v_t - 1 - C_C(1-u_t, 1-v_t; \gamma)
\]

where \( \gamma \in (-1, 1) \) in the Clayton copula and its rotation. However, according to Patton (2012), when the Clayton (and rotated Clayton) allows for negative dependence for \( \gamma \in (-1, 0) \), the form of this dependence is different from that of the positive dependence case (\( \gamma > 0 \)), and is not generally used in empirical work.

For the Clayton copula, the lower tail dependence \( \lambda_\gamma = 2-1/\gamma \) and the upper tail dependence \( \lambda_0 = 0 \). For the rotation of the Clayton copula, the upper tail dependence \( \lambda_0 = 2^{1/\gamma} \) and the lower tail dependence \( \lambda_\gamma = 0 \).

A.4. Gumbel copula and its rotation

The Gumbel copula is an extreme value copula that has higher probability concentrated in the upper tail. It is given by:

\[
C_G(u_t, v_t; \gamma) = \exp\left(-\left(-\log u_t\right)^{\gamma} + \left(-\log v_t\right)^{\gamma}\right)^{1/\gamma}
\]

and its rotation focuses on the lower tail dependence by:

\[
C_{CG}(u_t, v_t; \gamma) = u_t + v_t - 1 + C_G(1-u_t, 1-v_t; \gamma)
\]

where \( \gamma \in (1, \infty) \) in the Gumbel copula and its rotation. For the Gumbel copula, the upper tail dependence \( \lambda_0 = 2^{1/\gamma} \) and the lower tail dependence \( \lambda_\gamma = 0 \), whereas for the rotation of the Gumbel copula, the upper tail dependence is \( \lambda_\gamma = 0 \) and the lower tail dependence is \( \lambda_0 = 2^{1/\gamma} \).

A.5. Evolution of the copula parameters

The correlation coefficient \( \rho_t \) evolves according to the dynamic model proposed by Patton (2006):

\[
\rho_t = \Lambda \left(\Psi_0 + \Psi_1 \frac{10}{10} \sum_{j=1}^{10} \Phi^{-1}(u_{t-j}) \cdot \Phi^{-1}(v_{t-j}) + \Psi_2 \rho_{t-1}\right)
\]

where \( \Lambda \) denotes the logistic transformation \( \Lambda(x) = (1-e^{-x})(1+e^{-x})^{-1} \) that is used to keep \( \rho_t \) within \((-1, 1) \). For the Student-\( t \) copula, \( \Phi^{-1}(x) \) is substituted by \( t_{\nu,1}(x) \).

The dynamics of \( \tau_t (\tau_t = \gamma_t / (2 + \gamma_t)) \) of the Clayton copula and its rotation follow the evolution:

\[
\tau_t = \Lambda \left(\Psi_0 + \Psi_1 \frac{10}{10} \sum_{j=1}^{10} \left|u_{t-j} - v_{t-j}\right| + \Psi_2 \rho_{t-1}\right)
\]

where \( \Lambda \) denotes the logistic transformation \( \Lambda(x) = (1 + e^{-x})^{-1} \) to keep \( \tau_t \) in \((0, 1)\).

The parameter \( \gamma \) of the Gumbel copula and its rotation follows the following dynamics:

\[
\gamma_t = \Lambda \left(\Psi_0 + \Psi_1 \gamma_{t-1} + \Psi_2 \frac{10}{10} \sum_{i=1}^{10} \left|u_{t-i} - v_{t-i}\right|\right)
\]

where \( \Lambda \) denotes the logistic transformation \( \Lambda(x) = 1 + x^2 \) to keep the value of \( \gamma \) in \((1, \infty)\).

References


